# Series _ Electronic and optical semiconductor devices

## Series 8 _ The p-n junction in equilibrium

1-2. When the p- and n-type semiconductors are jointed together, the large carrier concentration gradients at the junction cause carrier diffusion. Holes from the p-side diffuse into the n-side, and electrons from the n-side diffuse into the p-side. As holes continue to leave the p-side, some of the negative acceptor ions ($N_A^-$) near the junction are left uncompensated, since the acceptors are fixed in the semiconductor lattice, whereas the holes are mobile. Similarly, some of the positive donor ions ($N_D^+$) near the junction are left uncompensated as the electrons leave the n-side. Consequently, a negative space charge forms near the p-side of the junction and a positive space charge forms near the n-side. This space charge region creates an electric field that is directed from the positive charge toward the negative charge (Figure 1a).

Figure 1b shows that the hole diffusion current flows from left to right, whereas the hole drift current due to the electric field flows from right to left. The electron diffusion current also flows from right to left, whereas the electron drift current flows in the opposite direction. Note that because of their negative charge, electrons diffuse from right to left, opposite the direction of electron current.

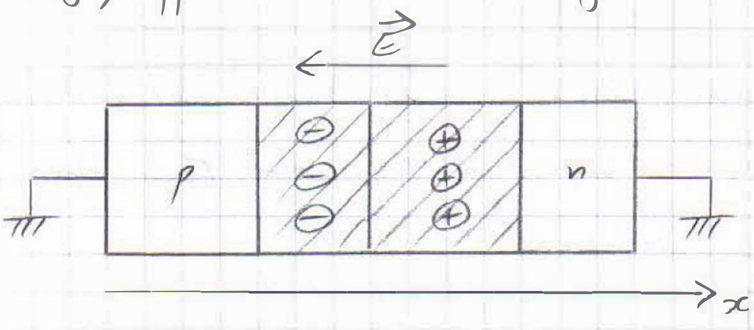



Fig. 1a

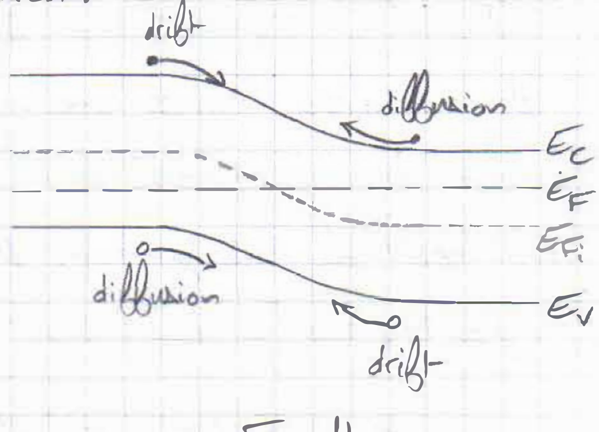



Fig. 1b

3_ At thermal equilibrium, that is the steady-state condition at a given temperature without any external excitations, the individual electron and hole currents flowing across the junction are identically zero. Thus, for each type of carrier the drift current due to the electric field must exactly cancel the diffusion current due to the concentration gradient.

4- Care has to be taken in order not to be mistaken between the number current densities (denoted with a lower case letter hereafter, $j$) and electrical current densities (denoted with an upper case letter, $J$). They are related through the relations:

$$J_n = -e j_n \quad \text{and} \quad J_p = e j_p$$

Starting with holes we thus have $\quad J_p = J_p\,(\text{drift}) + J_p\,(\text{diffusion})$

$$j_p\,(\text{drift}) = \mu_p p \mathcal{E}$$

$$j_p\,(\text{diffusion}) = -D_p \frac{dp}{dx} \qquad \text{with } D_p = \frac{k_B T}{e}\mu_p \quad (D_{p,n} \text{ are positive quantities})$$

↳ Einstein formula

and $\quad J_p = e\mu_p p \mathcal{E} - e D_p \frac{dp}{dx} = 0$

It is convenient to use the intrinsic Fermi level $E_{F_i}$ which is linked to the electric field through the relation $\mathcal{E} = \frac{1}{e}\frac{dE_{F_i}}{dx}$. Furthermore, we can define the related quantity $\phi$ as the electrostatic potential whose negative gradient equals the electric field: $\mathcal{E} = -\frac{d\phi}{dx}$

As a result we can write:

$$J_p = \mu_p p \frac{dE_{F_i}}{dx} - k_B T \mu_p \frac{dp}{dx} = 0 \quad (A)$$

We can also express the hole concentration as a function of the intrinsic carrier density $n_i$ and the difference between $E_{F_i}$ and $E_F$, the Fermi level of the junction. We thus obtain:

$$p = N_V \exp\left[-(E_F - E_V)/k_B T\right]$$

i.e. $p = N_V \exp\left[-(E_F - E_{F_i})/k_B T\right] \exp\left[-(E_{F_i} - E_V)/k_B T\right]$

so that $p = n_i \exp\left[(E_{F_i} - E_F)/k_B T\right]$

Its derivative will then write $\frac{dp}{dx} = \frac{p}{k_B T}\left[\frac{dE_{Fi}}{dx} - \frac{dE_F}{dx}\right]$

which is then substituted into (A) so that we get: $J_p = \mu_p p \frac{dE_F}{dx} = 0$

i.e. $\frac{dE_F}{dx} = 0$

Similarly, we obtain for the net electron current density:

$J_n = J_n\,(\text{drift}) + J_n\,(\text{diffusion})$

$j_n(\text{drift}) = -\mu_n n E$

$j_n(\text{diffusion}) = -D_n \frac{dn}{dx}$

and $J_n = e\mu_n n E + e D_n \frac{dn}{dx} = 0$

i.e. $J_n = \mu_n n \frac{dE_F}{dx} = 0$

Thus for the condition of zero net electron and hole currents, the Fermi level must be constant (i.e. independent of $x$) throughout the sample as illustrated in the energy band diagram of Fig. 1b.

It is worth pointing out that the carriers' flows shown in Fig. 1b are number current densities and not electrical current densities. For holes there is an equivalence between the two quantities (holes drift in the same direction as the electric field owing to their positive charge and for the diffusion part, the current has the same direction as holes) whereas it is not true for electrons (electrical current flows exhibit an opposite direction to the particle ones).

5_ The constant Fermi level required at thermal equilibrium results in a unique space charge distribution at the junction. The unique space charge distribution and the electrostatic potential $\phi$ are given by Poisson's equation:

$$\boxed{\frac{d^2\phi}{dx^2} = -\frac{dE}{dx} = -\frac{\rho(x)}{\varepsilon_r \varepsilon_0}}$$

Here we assume that all donors and acceptors are ionized whatever the position $x$ (i.e. also far from the junction). Consequently, the charge density due to donors, acceptors and free carriers is given by:

$$\boxed{\rho(x) = e\left[N_D(x) - N_A(x) + p(x) - n(x)\right]}$$

6- In regions far away from the metallurgical junction (i.e. the abrupt junction), charge neutrality is maintained and the total space charge density is zero. For these neutral regions we thus get $\frac{d^2\phi}{dx^2} = 0$ and

$N_D - N_A + p - n = 0$

For the p-type neutral region, we assume $N_D = 0$ and $p \gg n$ so that we obtain the relation $p - N_A = 0$, i.e. $p = N_A$. From question 4, we can then write $N_A = n_i \exp\left[(E_{F_i} - E_F)/k_B T\right]$. Moreover, $\phi_p = -\frac{1}{e}(E_{F_i} - E_F)\Big|_{x \leq -x_p}$

so that $N_A = n_i \exp\left[-e\phi_p / k_B T\right]$ which leads to $\boxed{\phi_p(x)\Big|_{x\leq -x_p} = -\frac{k_B T}{e}\ln\left(\frac{N_A}{n_i}\right)}$

limit of neutral p region

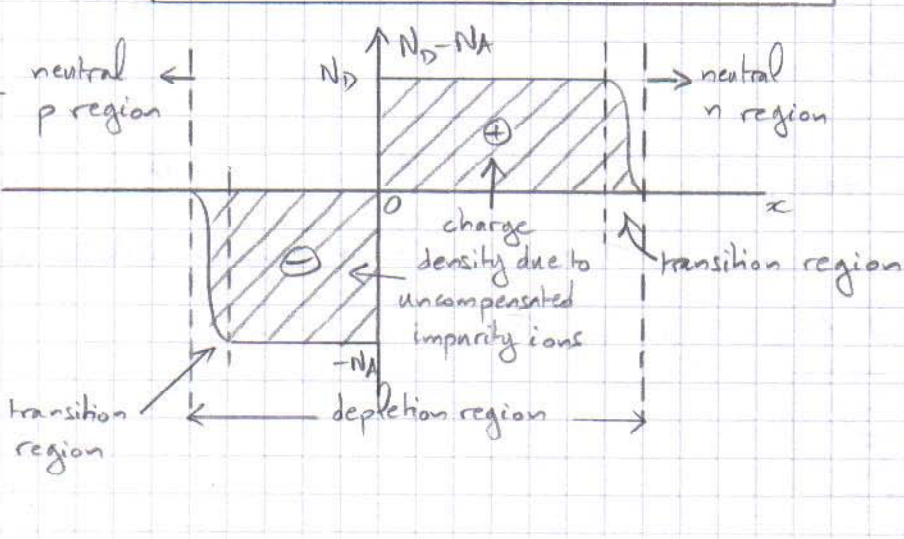
Similarly for the n-type neutral region we get:

$$\boxed{\phi_n(x)\Big|_{x \geq x_n} = \frac{k_B T}{e}\ln\left(\frac{N_D}{n_i}\right)}$$

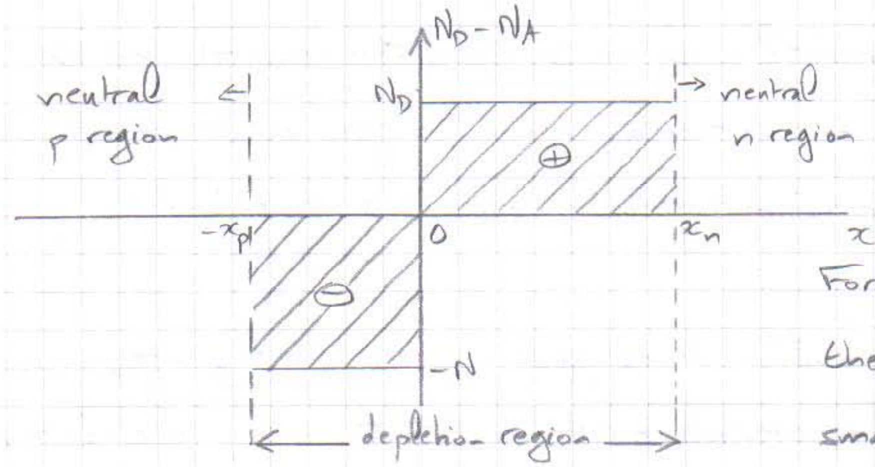
limit of neutral n region

7- The total electrostatic potential difference between the p-side and the n-side neutral regions at thermal equilibrium is called the built-in potential $V_{bi}$:

$$\boxed{V_{bi} = \phi_n - \phi_p = \frac{k_B T}{e}\ln\left(\frac{N_A N_D}{n_i^2}\right)}$$

8- 


a- Space charge distribution




b_ Rectangular approximation of the space charge distribution

For typical p-n junctions in Si or GaAs, the width of each transition region is small compared with the width of the depletion region so that we can neglect the transition region and represent the depletion region by the rectangular distribution shown above.

9_ In the depletion region, free carriers are totally depleted (i.e. $p = n = 0$) so that Poisson's equation simplified to $\dfrac{d^2\phi}{dx^2} = \dfrac{e}{\varepsilon_r \varepsilon_0}\left[N_D - N_A\right]$

For $-x_p \leq x < 0$, we have $\dfrac{d^2\phi}{dx^2} = \dfrac{e N_A}{\varepsilon_r \varepsilon_0}$

so that $\dfrac{d\phi}{dx} = \dfrac{e N_A}{\varepsilon_r \varepsilon_0}(x + x_p)$ (there is continuity in $x = -x_p$, the electric field is equal to zero)

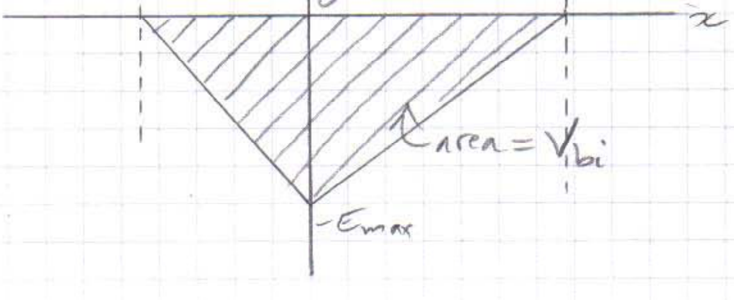
i.e. $$\boxed{\phi(x) = \phi_p^{neutral} + \frac{e N_A}{2\varepsilon_r \varepsilon_0}(x + x_p)^2} \qquad -x_p \leq x < 0$$

Similarly on the n-side of the depletion region we get

$$\boxed{\phi(x) = \phi_n^{neutral} - \frac{e N_D}{2\varepsilon_r \varepsilon_0}(x - x_n)^2} \qquad 0 < x \leq x_n$$

10_ The electric field is related to the electrostatic potential $\phi$ through the relation $\mathcal{E} = -\dfrac{d\phi}{dx}$ so that on the p-side, $\mathcal{E}(x) = -\dfrac{e N_A}{\varepsilon_r \varepsilon_0}(x + x_p)$ for $-x_p \leq x < 0$

and on the n-side, $\mathcal{E}(x) = \dfrac{e N_D}{\varepsilon_r \varepsilon_0}(x - x_n)$ for $0 < x \leq x_n$




$\mathcal{E}_{max}$, maximum field that exists at $x = 0$ and is given by

$$\mathcal{E}_{max} = \frac{e N_D}{\varepsilon_r \varepsilon_0} x_n = \frac{e N_A x_p}{\varepsilon_r \varepsilon_0}$$

11- The overall space charge neutrality of the semiconductor required that the total negative space charge per unit area in the p-side must precisely equal the total positive space charge per unit area in the n-side so that:

$$N_A x_p = N_D x_n$$

The total depletion layer width $W$ is given by $W = x_p + x_n$

By combining the results of previous questions (Q 7 to 9), we obtain

$$\phi(x=0^+) = \phi(x=0^-)$$

i.e. $$\phi_p^{neutral} + \frac{eN_A}{2\varepsilon_r\varepsilon_0} x_p^2 = \phi_n^{neutral} - \frac{eN_D}{2\varepsilon_r\varepsilon_0} x_n^2$$

which leads to $$V_{bi} = \frac{e}{2\varepsilon_r\varepsilon_0}\left[N_A x_p^2 + N_D x_n^2\right]$$

and as $x_p = \frac{N_D}{N_A} x_n$, $$V_{bi} = \frac{e}{2\varepsilon_r\varepsilon_0}\left[N_A \frac{N_D^2}{N_A^2} x_n^2 + N_D x_n^2\right]$$

$$\Rightarrow x_n = \left[\frac{2\varepsilon_r\varepsilon_0}{e}\left[\frac{N_A}{N_D^2 + N_D N_A}\right] V_{bi}\right]^{1/2}$$

and $$\boxed{W = \left[\frac{2\varepsilon_r\varepsilon_0 V_{bi}}{e}\right]^{1/2}\left(\left[\frac{N_A}{N_D^2 + N_D N_A}\right]^{1/2} + \left[\frac{N_D}{N_A^2 + N_D N_A}\right]^{1/2}\right)}$$

A more compact expression for $W$ is given by:

$$\boxed{W = \left[\frac{2\varepsilon_r\varepsilon_0}{e}\left(\frac{N_A + N_D}{N_A N_D}\right) V_{bi}\right]^{1/2}}$$

12- $V_{bi} = \frac{k_B T}{e} \ln\left(\frac{N_A N_D}{n_i^2}\right)$ $\qquad n_i \approx 1.45 \times 10^{10}\ cm^{-3}$, $T = 300K$

$V_{bi} \sim 0.873\,V$ and $W \sim 153.4\,nm$

$E_{max} \sim 113.8\ kV/cm$